National Technical University of Ukraine
«IGOR SIKORSKY KYIV
POLYTECHNIC INSTITUTE»

Department of Mathematical Physics and Differential Equations

HIGHER MATHEMATICS 3. MATHEMATICAL ANALYSIS
Working Program of the Academic Discipline (Syllabus)

Course Details	
Level of higher education	First (Bachelor's)
Branch of knowledge	G Engineering, Manufacturing and Construction
Specialty	G7 Automation, Computer-Integrated Technologies and Robotics
Educational program	Computer-Integrated Technologies and Robotics
Course status	Mandatory
Form of study	Full-time (day)
Year of study, semester	2nd year, autumn semester (3)
Course workload	**150 hours / 5 ECTS credits** Hours: Lectures **36**; Practical classes (seminars) **36**; Laboratory classes (computer practicum) **0**; Individual classes **0**; Self-study **78**
Semester control / control measures	Exam: **+**; Credit: **-**; MCW (specify number): **-**; CGW, CW, GW (specify number): **1**; HCW (specify number): **0**; Essay (specify number): **0**
Class schedule	On the university website: rozklad.kpi.ua
Language of instruction	Ukrainian
Information about the course leader / teachers	Lecturer: PhD (Physics and Mathematics), Associate Professor Olha Sulima olgasulimakpi@gmail.com Practical classes: PhD (Physics and Mathematics), Associate Professor Olha Sulima olgasulimakpi@gmail.com https://intellect.kpi.ua/profile/sov203
Course placement	https://ecampus.kpi.ua Department website, library information resources

Course Program

1. Course description, its aim, subject of study and learning outcomes

The aim of the course "Higher Mathematics 3. Mathematical Analysis" is to develop students' integral competence: the ability for logical thinking, analysis and synthesis, the formation of students' personality, the development of their intellect and abilities, the ability to solve complex specialized problems and practical issues of professional activity in the latest technologies, and to use methods of mathematical analysis in engineering calculations.

The course aims to provide students with the knowledge, skills and abilities needed to master professionally oriented disciplines and to give the necessary basic mathematical training for solving specialized problems of instrument engineering, for the development, improvement and operation of existing automation

systems using modern hardware, software and information technologies, and for theoretical research of automation objects.
Teaching mathematics involves:
– development of logical and algorithmic thinking;
– mastering the main methods of investigating and solving mathematical problems;
– mastering the main numerical methods of mathematics;
– the ability to apply mathematical knowledge independently and to carry out mathematical analysis of applied engineering problems.
The objective of studying the course is to develop the following competences:
GC 1. Ability to apply knowledge in practical situations.
GC 2. Ability to communicate in the state language both orally and in writing.
PC 1. Ability to apply knowledge of mathematics to the extent necessary to use mathematical methods for the analysis and synthesis of automation systems.
PC 3. Ability to analyse automation objects on the basis of knowledge of the processes occurring in them and to apply methods of automatic control theory for the investigation, analysis and synthesis of automatic control systems.
PC 4. Ability to apply methods of systems analysis, mathematical modelling, identification and numerical methods to develop mathematical models of individual elements and of automation systems as a whole, and to analyse the quality of their operation using the latest computer technologies.
Programme Learning Outcomes (PLO)
PLO 1. To know linear and vector algebra, differential and integral calculus, functions of several variables, functional series, differential equations for functions of one and several variables, operational calculus, the theory of functions of a complex variable, probability theory and mathematical statistics, and the theory of random processes to the extent necessary to use mathematical tools and methods in the field of automation.
PLO 6. To be able to apply methods of systems analysis, modelling, identification and numerical methods to develop mathematical and simulation models of individual elements and of automation systems as a whole, and to analyse the quality of their operation using the latest computer technologies.
PLO 12. To be able to use a variety of specialized software to solve typical engineering problems in the field of automation, in particular mathematical modelling, computer-aided design, database management and computer graphics methods.

2. Prerequisites and postrequisites of the course (place in the structural and logical scheme of training under the relevant educational program)

The course is taught in the 3rd semester on the basis of complete general secondary or vocational secondary education and the knowledge gained in the courses "Higher Mathematics 1. Analytic Geometry and Linear Algebra" and "Higher Mathematics 2. Differential Calculus".

3. Course content

Topic 1. Series.
Numerical series. Functional series. Power series. Fourier series.
Topic 2. Integral calculus of functions of several variables.
Double integrals. Triple integrals. Line integrals of the 1st and 2nd kinds. Surface integrals of the 1st and 2nd kinds.
Topic 3. Elements of field theory.
Scalar field. Directional derivative. Gradient. Vector field. Flux, circulation, divergence and curl of a vector field. The Hamilton operator.

4. Educational materials and resources

Main literature

1. Dubovyk V. P. Higher Mathematics / V. P. Dubovyk, I. I. Yuryk. – Kyiv: Vyshcha shkola, 2013. – 648 p. [in Ukrainian]
2. Kuzma O. V., Yatsiuk V. T. Multiple, Line and Surface Integrals. Fundamentals of Field Theory: teaching and methodological manual / O. V. Kuzma, V. T. Yatsiuk. – Kyiv: NTUU "KPI", 2016. – 113 p. Electronic edition approved by the Council of the Faculty of Physics and Mathematics on 24.06.16, minutes No. 6. – Electronic resource, file size 1.20 MB; access: ***http://ela.kpi.ua/handle/123456789/16675****. [in Ukrainian]*
3. Hrymaliuk V. P. Higher Mathematics: in 2 parts: textbook / V. P. Hrymaliuk, M. M. Kukharchuk, V. V. Yasinskyi. – Kyiv: Vipol, 2004. – Part 1. – 376 p. [in Ukrainian]
4. Dubovyk V. P. Higher Mathematics. Collection of Problems: textbook / V. P. Dubovyk, I. I. Yuryk. – Kyiv: A.S.K., 2005. – 648 p. [in Ukrainian]
5. Stryzhak T. H. Mathematical Analysis: Examples and Problems: textbook / T. H. Stryzhak, N. R. Konovalova. – Kyiv: Lybid, 1995. – 240 p. [in Ukrainian]

Additional literature

1. Herasymchuk V. S. Higher Mathematics. A Complete Course in Examples and Problems: in 3 parts: textbook / V. S. Herasymchuk, H. S. Vasylchenko, V. I. Kravtsov. – Kyiv: Knyhy Ukrainy LTD, 2009. – Part 1. – 578 p.; 2010. – Part 2. – 470 p.; 2009. – Part 3. – 400 p. [in Ukrainian]
2. Collection of Problems for Calculation Works in Higher Mathematics [Electronic resource]: textbook for students of specialty 141 "Electric Power Engineering, Electrical Engineering and Electromechanics" / Igor Sikorsky Kyiv Polytechnic Institute; compiled by A. L. Hrechko, M. Ye. Dudkin. – Electronic text data (1 file: 7.60 MB). – Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2021. – 280 p. [in Ukrainian]
3. Higher Mathematics. Elements of Field Theory and Theory of Series. Calculation Work [Electronic resource]: textbook for students of specialty 186 "Publishing and Printing" / Igor Sikorsky Kyiv Polytechnic Institute; compiled by O. I. Kushlyk-Dyvulska, N. V. Polishchuk. – Electronic text data (1 file: 2.27 MB). – Kyiv: Igor Sikorsky Kyiv Polytechnic Institute, 2018. – 110 p. [in Ukrainian]

Educational Content

5. Methods of mastering the course (educational component)

The main forms of learning are lectures, practical classes, individual assignments and students' independent work.

In lectures, students are taught the theoretical foundations of the course.

List of lectures

Topic 1. Series

Lecture 1. **Numerical series.**

1.1. Sum of a numerical series. Convergence and divergence of a numerical series.
1.2. Properties of numerical series. Necessary condition for the convergence of a series.
1.3. Harmonic series.
1.4. Series with positive terms. Cauchy integral test for convergence.
1.5. Sufficient tests for the convergence of series with positive terms: comparison test, limit comparison test.

Lecture 2. **Sufficient tests for the convergence of series with positive terms. Alternating numerical series.**

2.1. D'Alembert's ratio test.
2.2. Cauchy root test.
2.3. Alternating numerical series. Absolute and conditional convergence. Leibniz's theorem.

Lecture 3. **Functional series.**

3.1. Convergence and divergence of functional series.
3.2. Uniform convergence of functional series. Cauchy criterion for uniform convergence.
3.3. Weierstrass test for uniform convergence.

3.4. Properties of uniformly convergent functional series: continuity, differentiation and integration.

Lecture 4. **Power series. Taylor and Maclaurin series.**

4.1. Interval of convergence of a power series. Abel's theorem.

4.2. Radius of convergence of a power series. The Cauchy–Hadamard theorem.

4.3. Properties of power series. Differentiation and integration of power series.

4.4. Basic concepts.

4.5. Criterion for the convergence of a Taylor series.

4.6. Binomial series. Maclaurin series expansion of elementary functions:

$$y = e^x;\ y = \sin x;\ y = \cos x;\ y = \ln(1 + x);\ y = \arctan x.$$

Lecture 5. **Approximate calculations using power series.**

5.1. Integration of differential equations using power series.

5.2. Approximate calculation of logarithms.

5.3. Approximate calculation of function values.

5.4. Approximate calculation of definite integrals.

Lecture 6. **Fourier series.**

6.1. Trigonometric series. Properties of trigonometric series.

6.2. Fourier series. Fourier coefficients.

6.3. Dirichlet's theorem on a sufficient condition for expanding a function with period 2π into a Fourier series.

6.4. Fourier series for even and odd functions with period 2π.

Lecture 7. **Trigonometric Fourier series for functions of arbitrary period.**

7.1. Fourier series for functions with period *2l*. Fourier series for even and odd functions with period *2l*.

7.2. Fourier cosine series expansion of a function defined on [0; *l*].

7.3. Fourier sine series expansion of a function defined on [0; *l*].

7.4. Fourier series for functions defined on the interval [0; *l*].

7.5. Complex form of the Fourier series.

Topic 2. Integral calculus of functions of several variables

Lecture 8. **Double integral. Applications of the double integral.**

8.1. Definition of the double integral. The problem of the volume of a cylindrical body.

8.2. Conditions for the existence of the double integral. Integrability criterion.

8.3. Properties of double integrals.

8.4. Calculation of double integrals. Region D is a rectangle; region D is a curvilinear region.

8.5. Change of variables in the double integral. The double integral in polar and generalized polar coordinates.

8.6. Calculation of the area of a plane figure.

8.7. Calculation of the volume of a solid.

8.8. Calculation of surface area.

8.9. Applications of double integrals to problems of mechanics.

Lecture 9. **Triple integral.**

9.1. The problem of the mass of a solid. Definition of the triple integral and its properties.

9.2. Conditions for the existence of triple integrals. Integrability criterion.

9.3. Calculation of triple integrals.

9.4. Change of variables in the triple integral. The triple integral in cylindrical, spherical and generalized spherical coordinates.

9.5. Calculation of the volume of a solid. Applications of the triple integral to problems of mechanics.

Lecture 10. **Line integrals of the 1st kind (with respect to arc length).**

10.1. Definition of the line integral of the 1st kind.

10.2. Properties of the line integral with respect to arc length.

10.3. Calculation of the line integral of the 1st kind for different ways of defining a curve.

10.4. Applications of line integrals of the 1st kind.

Lecture 11. **Line integrals of the 2nd kind (with respect to the coordinates of points on a curve).**

11.1. The problem of the work done by a variable force moving a material point along a curve.

11.2. Definition of the line integral of the 2nd kind.

11.3. Main properties of line integrals of the 2nd kind.

11.4. Sufficient conditions for the existence of line integrals of the 2nd kind.

11.5. Calculation of the line integral of the 2nd kind for different ways of defining a curve.

11.6. Relationship between line integrals of the 1st and 2nd kinds.

Lecture 12. **Conditions for the independence of the line integral of the 2nd kind from the path of integration.**

12.1. Green's formula and its application to calculating the area of a plane figure.

12.2. Conditions for the independence of the line integral of the 2nd kind from the path of integration.

12.3. Integration of total differentials.

12.4. Applications of line integrals of the 2nd kind.

Lecture 13. **Surface integrals of the 1st kind (with respect to surface area).**

13.1. Definition of the surface integral of the 1st kind.

13.2. Conditions for the existence and calculation of surface integrals of the 1st kind.

13.3. Applications of surface integrals of the 1st kind to problems of geometry and mechanics.

Lecture 14. **Surface integrals of the 2nd kind (with respect to coordinates).**

14.1. Orientation of a surface.

14.2. Definition of surface integrals of the 2nd kind and their properties.

14.3. Conditions for the existence and calculation of surface integrals of the 2nd kind.

14.4. Relationship between surface integrals of the 1st and 2nd kinds.

14.5. The Gauss–Ostrogradsky formula.

14.6. Stokes' formula.

Topic 3. Elements of field theory

Lecture 15. **Elements of field theory.**

15.1. Scalar field. Directional derivative. Gradient and its properties.

15.2. Vector field. Vector lines. Flux of a vector field through a surface and its calculation by various methods.

15.3. Divergence of a vector field. The Gauss–Ostrogradsky formula in vector form.

15.4. Circulation of a vector field. Curl of a vector field. Stokes' formula in vector form.

15.5. Special fields: potential, solenoidal, harmonic.

15.6. The Hamilton operator.

In practical classes – assignments (according to the list of main literature given above)

List of practical classes

Practical class 1. Sufficient tests for the convergence of numerical series with positive terms. Investigating numerical series with positive terms for convergence.

Practical class 2. Alternating numerical series. Investigating conditional and absolute convergence.

Practical class 3. Functional series. Weierstrass's theorem on uniform and absolute convergence. Investigating a functional series for uniform convergence.

Practical class 4. Finding the region of convergence of power series.

Applications of power series. Maclaurin series expansion of elementary functions.

Practical class 5. Approximate calculation of function values and integrals. Finding solutions of differential equations.

Practical class 6. Fourier series expansion of a function on the interval $[-\pi, \pi]$.

Practical class 7. Fourier series expansion of periodic functions with arbitrary period *2l*. Test "Series".

Practical class 8. Calculation of double integrals. Change of variables in the double integral. Calculation of double integrals in polar coordinates.

Practical class 9. Applications of double integrals.

Calculation of triple integrals. Change of variables in the triple integral: calculation in cylindrical and spherical coordinates.

Practical class 10. Applications of the triple integral. Control work on the topic "Multiple Integrals".

Practical class 11. Calculation of line integrals of the 1st kind. Applications of line integrals of the 1st kind.

Practical class 12. Calculation of line integrals of the 2nd kind. Independence of the line integral of the 2nd kind from the path of integration. Applications of line integrals of the 2nd kind.

Practical class 13. Calculation of surface integrals of the 1st kind.

Calculation of surface integrals of the 2nd kind. Relationship between surface integrals of the 1st and 2nd kinds. Applications of surface integrals.

Practical class 14. Elements of field theory: finding the directional derivative and the gradient of a scalar field. Calculation of the flux of a vector field through a surface. Control work on the topic "Flux of a Vector Field through a Surface".

Practical class 15. Calculation of the circulation, divergence and curl of a vector field. Checking a field for potentiality. Test "Elements of Field Theory".

6. Student's independent work

Students' independent work includes consolidating and deepening knowledge from the lecture materials, independent study of selected topics using the recommended teaching materials, preparation for practical classes, completion of a home control work, and completion of the calculation and graphic work (divided into two parts according to the scheduled semester attestations).

Policy and Control

7. Course policy (educational component)

The following requirements apply to students:

- ***attendance rules:***
  - *do not be late for classes; if more than 15 minutes late, enter in the second half of the class so as not to distract others; notify the teacher in advance about missing a class for a valid reason or due to illness (confirm with a copy of a medical certificate);*
  - *in distance learning mode: Zoom conference via the teacher's link;*
- ***rules of conduct in class:***
  - *engaging in any activity not directly related to the course is prohibited;*
  - *the use of communication devices to search the Internet or the Moodle distance learning platform for information needed to complete assignments is allowed;*
  - *disrupting classes in any way is prohibited;*
- ***rules for awarding bonus and penalty points:***
  - *penalty points are given for late completion and defence of the CGW; bonus points are given for activity in practical classes;*
  - *the maximum number of bonus and penalty points is determined in accordance with the Regulations on the System for Assessing Learning Outcomes at Igor Sikorsky Kyiv Polytechnic Institute, the Regulations on Current, Calendar and Semester Control of Learning Outcomes at Igor Sikorsky Kyiv Polytechnic Institute, and other regulations and recommendations in force at Igor Sikorsky Kyiv Polytechnic Institute;*
- ***assessment policy for control measures:***
  - *control measures are assessed in accordance with the Regulations on the System for Assessing Learning Outcomes at Igor Sikorsky Kyiv Polytechnic Institute, the Regulations on Current, Calendar and Semester Control of Learning Outcomes at Igor Sikorsky Kyiv*

Polytechnic Institute, and other regulations and recommendations in force at Igor Sikorsky Kyiv Polytechnic Institute;

- *the lower limit of a positive assessment for each control measure must be at least 60% of the points allocated to that control measure;*
- *a negative result is assessed as 0 points.*

Academic Integrity

The policy and principles of academic integrity are defined in Section 3 of the Code of Honour of Igor Sikorsky Kyiv Polytechnic Institute. More details: https://kpi.ua/code.

Norms of Ethical Behaviour

The norms of ethical behaviour of students and staff are defined in Section 2 of the Code of Honour of Igor Sikorsky Kyiv Polytechnic Institute. More details: https://kpi.ua/code.

Appeals against Control Measure Results

Students have the right to raise any issue related to the procedure for conducting or assessing control measures and to expect it to be considered by a commission.

8. Types of control and the rating system for assessing learning outcomes (RSA)

Current control (maximum score – 50 points):

– surveys in practical classes;
– completion of thematic control works (TCW 1, TCW 2);
– writing the modular control work (MCW parts 1, 2);
– completion and defence of the calculation and graphic work (CGW parts 1, 2);

Table of correspondence between rating points and types of current control.

Type of current control	*Maximum possible number of points*
TCW 1, TCW 2	10 points (5+5)
MCW (parts 1, 2)	20 points (10+10)
CGW (parts 1, 2)	20 points (10+10)

Scoring criteria:

1. Completion of the thematic control work.

Thematic control works (TCW 1, TCW 2) are conducted on the topics of the section "Elements of Linear Algebra and Analytic Geometry".

"excellent" – 5 points – The tasks are completed correctly and contain all the necessary calculations and explanations.

"good" – 4–4.5 points – The tasks are completed correctly, but there are shortcomings. In particular, correct approaches to the solution are used, but explanations are completely missing, there are calculation errors, etc. However, these shortcomings indicate insufficient attention while completing the task rather than a lack of knowledge and skills.

"satisfactory" – 3–3.5 points – The tasks are completed partially or with some errors. In particular, a formula is written incorrectly, which leads to a wrong answer, or there are significant errors in transforming analytical expressions. However, the generally correct approach to solving the tasks is shown.

"unsatisfactory" – 0–2.5 points – The tasks are completed incorrectly or with significant errors (less than 60% of the meaningful steps are performed correctly).

2. Completion of the modular control work.

The modular control work (MCW) consists of two parts and covers the topics of the sections "Elements of Linear Algebra and Analytic Geometry" and "Differential Calculus of a Function of One Variable". The modular control work contains a theoretical and a practical part.

"excellent" – 9.5–10 points – The answer to the theoretical questions is complete and correct and contains the necessary explanations, analytical expressions and graphs. The practical tasks are completed correctly and contain all the necessary calculations and explanations.

"good" – 7.5–9 points – The answer to the theoretical questions is sufficiently complete and correct and contains the necessary explanations, analytical expressions and graphs. However, the analytical expressions do not include explanations of their components, the graphs lack the necessary labels, and the answer is not illustrated with an example. The practical tasks are completed correctly and correct approaches to the solution are used, but explanations are completely missing, there are calculation errors, etc. Thus, the tasks are not complete enough, but at least 75% of the essential content is presented.

"satisfactory" – 6–7 points – The answer to the theoretical questions is correct but incomplete. Some necessary analytical expressions or graphs are missing. The answer contains a significant amount of information that corresponds to the general topic of the question but does not specify the aspects stated in the question itself. The practical tasks are completed partially or with some errors. In particular, a formula is written incorrectly, which leads to a wrong answer, or there are significant errors in transforming analytical expressions. However, the generally correct approach to solving the tasks is shown. The answer is at least 60% complete in terms of essential content.

"unsatisfactory" – 0–5 points – The answer to the theoretical questions contains less than 60% of correct and meaningful information. Key explanations and important analytical expressions are missing. The practical tasks are completed incorrectly or with significant errors (less than 60% of the meaningful steps are performed correctly).

3. Completion of the calculation and graphic work.

The calculation and graphic work (CGW) consists of two parts and covers the topics of the sections "Introduction to Mathematical Analysis" and "Differential Calculus of a Function of One Variable". A defence of the calculation and graphic work is required. A student is admitted to the defence provided that all tasks are completed correctly, with possible minor shortcomings and errors.

"excellent" – 9.5–10 points – The tasks are completed correctly and contain all the necessary calculations and explanations.

"good" – 7.5–9 points – The tasks are completed correctly and correct approaches to the solution are used, but explanations are completely missing, there are calculation errors, etc. Thus, the tasks are not complete enough, but at least 75% of the essential content is presented.

"satisfactory" – 6–7 points – The tasks are completed partially or with some errors. In particular, a formula is written incorrectly, which leads to a wrong answer, or there are significant errors in transforming analytical expressions. However, the generally correct approach to solving the tasks is shown. The answer is at least 60% complete in terms of essential content.

"unsatisfactory" – 0–5 points – The tasks are completed incorrectly or with significant errors (less than 60% of the meaningful steps are performed correctly).

Calendar control is carried out twice a semester to monitor the current status of meeting the syllabus requirements.

Table of correspondence between attestation dates and the conditions for receiving attestation.

Attestation date	*Conditions for receiving attestation*	
	Current rating	*Current control measure*

Week 7	≥ 50% of the maximum possible number of points at that moment	TCW 1, TCW 2 MCW (part 1)
Week 13	≥ 50% of the maximum possible number of points at that moment	TCW 1, TCW 2 MCW (parts 1, 2) CGW (parts 1, 2)

Semester control: exam (maximum score – 50 points).

The exam paper consists of two theoretical questions and three practical tasks. The number of points a student can receive for answering a theoretical question or solving a practical task:

– a complete answer to a theoretical question or a correctly solved practical task – 10 points;

– minor errors in the answer to a theoretical question or in solving a practical task – 6 points;

– an incomplete answer to a theoretical question or errors in solving a practical task – 3 points;

– no answer to a theoretical question or the practical task is not completed – 0 points.

A student's rating for the credit module is determined on a 100-point scale and consists of the points received for all types of current control and the points received at the exam.

Current control: express surveys, surveys on the topic of the class, writing control works, test.

Calendar control: carried out twice a semester to monitor the current status of meeting the syllabus requirements.

Semester control: exam.

Conditions for admission to semester control: a minimum positive grade for the thematic control work and the modular control work, and credit for the calculation and graphic work. The semester rating is at least 36 points.

Exam retakes take place within the period set by the dean's office.

Table of correspondence between rating points and grades on the university scale:

Number of points	*Grade*
100-95	Excellent
94-85	Very good
84-75	Good
74-65	Satisfactory
64-60	Sufficient
Less than 60	Unsatisfactory
Admission conditions not met	Not admitted

9. Additional information on the course (educational component)

List of questions for semester control

1. Numerical series. Sum of a numerical series. Convergence and divergence of a numerical series.

2. Properties of convergent numerical series.

3. Necessary condition for the convergence of a numerical series.

4. Harmonic series $\sum_{n=1}^{\infty} 1/n$.

5. Sufficient tests for the convergence of series with positive terms:

- *first and second comparison tests;*

- *D'Alembert's ratio test;*
- *Cauchy root test;*
- *Cauchy integral test; investigate the convergence of* $\sum_{n=1}^{\infty} 1/n^{\alpha}$.

6. Alternating numerical series. Leibniz's theorem. Absolute and conditional convergence.

7. Functional series. Weierstrass test for uniform convergence.

8. Power series. Abel's theorem.

9. Interval of convergence of a power series. Calculation of the radius of convergence of a power series.

10. Integration and differentiation of power series.

11. Taylor and Maclaurin series. Binomial series. Maclaurin series expansion of elementary functions:

$$y = e^x;\ y = \sin x;\ y = \cos x;\ y = \ln(1 + x);\ y = \arctan x.$$

12. Trigonometric series. Properties of trigonometric series.

13. Fourier series. Fourier coefficients.

14. Dirichlet's theorem on a sufficient condition for expanding a function with period 2π into a Fourier series.

15. Fourier series for even and odd periodic functions with period 2π.

16. Fourier series for functions with period 2l. Fourier series for even and odd functions with period 2l.

17. Fourier cosine series expansion of a function defined on [0; l].

18. Fourier sine series expansion of a function defined on [0; l]. Fourier series for functions defined on the interval [0; l].

19. Definition of the double integral. Existence theorem for double integrals. Geometric meaning.

20. Calculation of double integrals:

- *region D is a rectangle;*
- *region D is a curvilinear region.*

21. Properties of double integrals.

22. Change of variables in the double integral. The double integral in polar coordinates.

23. Applications of double integrals:

- *area of a plane figure;*
- *volume of a solid;*
- *surface area;*
- *mass of a plate, centre of mass of a plate, static moments, moments of inertia of a plate.*

24. Definition of the triple integral. Existence theorem for triple integrals.

25. Calculation of triple integrals.

26. Change of variables in the triple integral. The triple integral in cylindrical, spherical and generalized spherical coordinates.

27. Applications of triple integrals:

- *calculation of the volume of a solid;*
- *mass of a solid, moments of inertia of a solid about the coordinate axes, moments of inertia of a solid about the coordinate planes, static moments of a solid about the coordinate planes, coordinates of the centre of mass of a solid.*

28. Definition of the line integral of the 1st kind (with respect to arc length).

29. Calculation of line integrals of the 1st kind. The curve AB is given:

- *by parametric equations* $x = x(t);\ y = y(t);\ t \in [c;\ d]$;
- *by the equation* $y = f(x),\ x \in [a;\ b]$ *(in Cartesian coordinates);*
- *by the equation* $\rho = \rho(\varphi),\ \varphi \in [\alpha;\ \beta]$ *(in polar coordinates).*

30. Applications of line integrals of the 1st kind: surface area, arc length of a curve, mass of a curve, static moments of a curve about the coordinate axes, moments of inertia of a curve about the coordinate axes, coordinates of the centre of mass of a curve.

31. Definition of the line integral of the 2nd kind (with respect to the coordinates of points on a curve).

32. Main properties of line integrals of the 2nd kind.

33. Calculation of line integrals of the 2nd kind:

- *the curve AB is given by parametric equations* $x = x(t);\ y = y(t);\ t \in [c;\ d]$;
- *the curve AB is given by an equation in Cartesian coordinates:* $y = f(x),\ x \in [a;\ b]$.

34. Green's formula.

35. Conditions for the independence of line integrals of the 2nd kind from the path of integration.
36. Applications of line integrals of the 2nd kind: calculation of the area of a plane figure.
37. Calculation of the work done by a plane force along a curve AB using the line integral of the 2nd kind.
38. Surface integrals of the 1st kind. Applications of surface integrals of the 1st kind:

- *mass of a surface;*
- *static moments of a surface about the coordinate planes;*
- *coordinates of the centre of mass of a surface;*
- *moments of inertia of surfaces about the coordinate axes.*

39. Surface integrals of the 2nd kind. Relationship between surface integrals of the 1st and 2nd kinds. The Gauss–Ostrogradsky formula. Stokes' formula.
40. Elements of field theory. Scalar field. Directional derivative. Gradient and its properties.
41. Vector field. Vector lines. Flux of a vector field through a surface.
42. Circulation of a vector field.
43. The Ostrogradsky–Gauss formula. Divergence of a vector field.
44. Curl of a vector field. Stokes' formula. The Hamilton operator.
45. Special fields: potential, solenoidal, harmonic.

Working program of the academic discipline (syllabus):
Compiled by
Associate Professor of the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics, PhD (Physics and Mathematics), Associate Professor Olha Sulima
Approved *by the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics (minutes No. 10 of 24.06.2026)*
Agreed *by the Methodical Commission of the Faculty of Robotics and Instrument Engineering (minutes No. 6/26 of 26.06.2026)*